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UDC 662.611:662.62:66.096.5
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The distribution of fuel particles in a bed over the length of the combustion chamber is analytically investigated. It is shown that point loading is inappropriate as a means of ensuring normal operation of the combustion chamber.

Combustion of solid fuels in a so-called low-temperature fluidized bed is undoubtedly a progressive method of fuel combustion in energy steam generators. However, its use is associated with great difficulties; one of the main difficulties is the loading of coal into the fluidized bed. In most large experimental and pilot plants, use of the "American" system has been predominant recently; that is, delivery of coal by pneumotransport through a tube above the gas-distributor grid. Experience of boiler startup in Riversville [1] has shown that this system is extremely unwieldy, since point introduction of coal always associated with nonuniformity of combustion over the bed cross section and reducing this nonuniformity to an acceptable level requires the use of an enormous number of delivery points (approximately one input per $\mathrm{m}^{2}$ area of the gas-distributor grid, which causes another difficulty: uniform distribution of the coal over all the delivery points.

This situation will now be evaluated analytically, beginning with the "plane" case, when the fuel is continuously loaded in the bed at $x=0$ and moves by mixing with inert material in the direction of the $x$ axis. It is assumed that the fuel concentration in the bed is small, and all the ashes are carried off with the gas; then the convective flux of particles in the horizontal direction may be neglected. If in the first approximation the diffusional model of mixing in the fluidized bed is taken as the basis (this is acceptable for a rough estimate), the distribution of the fuel concentration $C$ in the bed may be described by the equation

$$
\begin{equation*}
h D \frac{d^{2} C}{d x^{2}}-f(C)=0 \tag{1}
\end{equation*}
$$

Here $f(C)$ is understood to mean the amount of carbon (it is assumed that there are no other elements in the fuel) converted to CO and $\mathrm{CO}_{2}$ per unit area of the gas-distributor grid as a result of combustion and gasification, $\mathrm{kg} / \mathrm{m}^{2} \cdot \mathrm{sec}$.

In the absence of longitudinal convective transfer of particles, the boundary conditions are written in the following form

$$
\begin{equation*}
x=0, j=-D h \frac{d C}{d x}, x=l, \frac{d C}{d x}=0 \tag{2}
\end{equation*}
$$

where the flux $j$ of fuel fed to the bed is referred to unit width of the bed perpendicular to the x axis.

To determine $f(C)$, it is expedient to use the results of simple experiments on periodicaction apparatus, where it is possible, after one experiment, to obtain information currently of interest: the dependence of the combustion rate on the concentration of coal, varying with progressive burnup of the initial amount fluidized together with a sufficient amount of inert material.

It is experimentally simpler to determine not the concentration $C$ of fuel in unit volume but its relative content $z$ in the bed, burning samples removed from the bed at different times. Assuming, for simplicity, that the density of the fluidized bed $p$ is constant, it is possible to write the relation: $z=C / p$. The function $f(z)$, equal to the amount of fuel burnup (more)
S. M. Kirov Ural Polytechnic Institute, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 42, No. 1, pp. 122-129, January, 1982. Original article submitted September 30, 1980.
precisely, gasifying) in unit time per unit area of the gas-distributor grid, may be found from the composition of the gases released from the bed. The product $f(z) V_{0}$ is the air flow rate theoretically necessary for the burnup of this amount of fuel (in normal conditions); w is the air flow rate actually passed through unit area of grid in unit time (the velocity is also referred to normal conditions). The excess-air coefficient $\alpha(z)=w /\left(f(z) V_{0}\right)$; hence $f(z)=$ $w /\left(\alpha(z) V_{0}\right.$. The quantity $\alpha(z)$ is determined from the gas composition [2]

$$
\begin{equation*}
\alpha(z)=\frac{1}{1-\frac{79}{21} \frac{\mathrm{O}_{2}-\left(0.5 \mathrm{CO}+0.5 \mathrm{H}_{2}+2 \mathrm{CH}_{4}\right)}{2}} \tag{3}
\end{equation*}
$$

The experiments were conducted on a laboratory apparatus with a cylindrical combustion chamber of internal diameter 0.15 m . In the lower part of the combustion chambers, a domed gas-distributor grid was placed, and beneath this air was supplied from a compressor. For preliminary heating of the apparatus, an electrical heater wound on the body of the combustion chamber was used. Semicoke of Irsha-Borodinskii lignite was burned, of elementary composition (\% of the working mass): $C=75.55 ; \mathrm{H}=1.51 ; 0=5.64 ; \mathrm{N}=0.81 ; \mathrm{S}=0.1$. The experiments were conducted in a periodic mode: a batch of semicoke (mean particle size 400 mm) was loaded in a mixture with corundum ( $120 \mu \mathrm{~m}$ ) and completely burned up. The height of the initial charge was 0.25 m , the air velocity at the cross section of the empty apparatus was $0.38 \mathrm{~m} / \mathrm{sec}$ at a bed temperature $700^{\circ} \mathrm{C}$. For analysis of the gases released, an LKhM-8MD chromatograph was used. In the course of fuel burnup from the bed, samples were removed by means of a special probe and then analyzed for fuel content. The probe/sampling device was a complete cylinder of glass enclosed by a cover. Values of the fuel content in the bed, the composition of the gases released with progressive burnup of the fuel batch, and the air flow-rate coefficients $\alpha(z)$ are given in Table 1.

Setting

$$
\begin{equation*}
f(z)=\frac{w}{\alpha(z) V_{0}}=\frac{2 w}{V_{0}} \frac{1}{2 \alpha(z)}=\frac{2 w}{V_{0}} \psi(z) \tag{4}
\end{equation*}
$$

the limits of variation of the dimensionless quantity $\psi(z)=1 /(2 \alpha(z))$ will now be considered. As $z \rightarrow 0$, the combustion products will obviously contain not only nitrogen but also oxygen, $\alpha(z) \rightarrow \infty$ and $\psi(z) \rightarrow 0$. In the other limiting case, when $z \rightarrow 1$, the gas forming in the burnup of pure carbon may contain $\mathrm{N}_{2}, \mathrm{O}_{2}, \mathrm{CO}$, and $\mathrm{CO}_{2}$; at infinitely large reaction rates of the oxidation of carbon and the reduction of $\mathrm{CO}_{2}$, the gas may contain only $\mathrm{N}_{2}$ and CO . In this case, in the burnup of pure carbon the $C 0$ content will be ( $21 / 0.605$ ) \% and the $\mathrm{N}_{2}$ content will be $(100-\mathrm{CO}) \%$.

As a result, $\alpha(z)$ in Eq. (3) is found to be 0.5 and $\psi(z)=1$. Hence, theoretically, $\psi(z)$ may vary from 0 to 1 , and practically within narrower limits because gas containing only CO and $\mathrm{N}_{2}$ is never formed in a low-temperature fluidized bed. According to the experimental results, a dependence for $\psi(z)$ for semicoke of Irsha-Borodinskii lignite was constructed; this dependence was well approximated by a function of the form

$$
\begin{equation*}
\psi(z)=0.65(1-\exp (-34 z)) \tag{5}
\end{equation*}
$$

Reducing Eq. (1) and the boundary conditions in Eq. (2) to dimensionless form, it is found that

$$
\frac{d^{2} z}{d x^{2}}-\frac{2}{\tilde{D}} \Psi(z)=0
$$

when

$$
\tilde{x}=0 \quad \frac{d z}{d \tilde{x}}=-\frac{1}{\tilde{D} \alpha} ; \text { when } \tilde{x}=1, \frac{d z}{d \tilde{x}}=0
$$

Here $z=C / \rho, \tilde{x}=x / Z, \tilde{D}=\rho D h V_{0} / w Z^{2}$. In addition, the fuel concentration at $\tilde{x}=0$ is denoted by $z_{0}$ and that at $\tilde{x}=1$ by $z_{\eta}$. The solution of the system takes the form

$$
\begin{equation*}
\int_{z_{0}}^{z} \frac{d z}{\left(\frac{1}{\tilde{D} \bar{\alpha}}\right)^{2}+\frac{4}{\tilde{D}} \int_{z_{0}}^{z} \psi(z) d z}= \pm \vec{x} \tag{6}
\end{equation*}
$$

TABLE 1. Change in Fuel Content in the Bed and Composition of the Gases Released with Progressive Burnup of the Fuel Batch

| $z$ | $\mathrm{O}_{2}$, vol. \% | $\mathrm{CO}_{2}$, vol. \% | $\mathrm{CO}, \mathrm{vol} . \%$ | $\alpha$ |
| :---: | :---: | :---: | :---: | :---: |
| 0,128 | 0,08 | 14,03 | 11,53 | 0,78 |
| 0,101 | 0,07 | 15,34 | 9,96 | 0,80 |
| 0,061 | 0,26 | 15,88 | 7,46 | 0,85 |
| 0,040 | 1,56 | 17,83 | 2,74 | 1,01 |
| 0,025 | 4,50 | 16,09 | 0,02 | 1,27 |
| 0,016 | 9,30 | 11,39 | 0 | 1,79 |

(From physical considerations, only the minus sign is retained in Eq. (6).)
Since

$$
\begin{equation*}
\frac{d z}{d \widetilde{x}}=-\sqrt{\left(\frac{1}{\tilde{D} \bar{\alpha}}\right)^{2}+\frac{4}{\tilde{D}} \int_{z_{0}}^{z} \Psi(z) d z} \tag{7}
\end{equation*}
$$

it follows, taking Eq. (2') into account, that

$$
\begin{equation*}
\left(\frac{1}{\tilde{D} \alpha}\right)^{2}=-\frac{4}{\tilde{D}} \int_{z_{0}}^{z_{0}} \psi(z) d z \tag{8}
\end{equation*}
$$

Substituting Eq. (8) into Eq. (6), Eq. (6) may be written in more compact form

$$
\begin{equation*}
\int_{z_{0}}^{z} \frac{d z}{\sqrt{\frac{1}{D}} \int_{z_{l}}^{z} \psi(z) d z}=-\tilde{x} \tag{9}
\end{equation*}
$$

When $\tilde{\mathbf{x}}=1$

$$
\begin{equation*}
\int_{z_{0}}^{z_{l}} \frac{d z}{\frac{1}{\tilde{D}} \int_{z_{l}}^{z} \psi(z) d z}=-1 \tag{10}
\end{equation*}
$$

Equations (8) and (10) define the limiting concentrations $z_{0}$ and 2 , while Eq. (9) - or Eq. (6) - defines the concentration distribution over the length of the chamber. The amount of fuel burnup in unit time in a chamber of length $Z$ is

$$
\begin{equation*}
j=\int_{0}^{l} f(z(x)) d x \tag{11}
\end{equation*}
$$

The flux of fuel at point $j$ may be expressed in terms of the coefficients most commonly used in energetics: the air flow-rate coefficient $\bar{\alpha}$ (averaged over the chamber) and the theoretically necessary volume of air $V_{0}$ (in normal conditions) for the burnup of 1 kg of fuel. The amount of air theoretically necessary for the burnup of all the fuel supplied is $\bar{\alpha} j V_{o}$, while the amount of air actually supplied under the gas-distributor grid is w $\mathcal{Z}$, where w is the fluidization rate reduced to normal conditions. Then it follows that

$$
\begin{equation*}
j=\frac{w l}{V_{0} \alpha} \tag{12}
\end{equation*}
$$

In order to reduce the heat lost with the gas released, it is desirable for the excessair coefficient $\bar{\alpha}$ to be not greatly more than unity (in energy boilers, $\alpha=1.1-1.2$ ). In this case, in cross sections close to the air input, the value of $\alpha$ will be less than unity (the combustion products released from the bed contain $C O$ ), while in remote cross sections $\alpha$ will be larger than unity. If it assumed that the combustion products released from the bed do not burn above it, because of the absence of longitudinal mixing, the presence of CO leads to the appearance of chemically incomplete combustion, despite the excess of air in the remote bed cross sections.

The magnitude of the chemically incomplete combustion will now be estimated. The quantity $q_{3}$ may be calculated either directly, knowing from experiment the content of in-complete-combustion products in the gases released from a bed with a given fuel concentration or from the excess-air coefficient. In the latter case, assuming for simplicity that the fuel consists only of carbon ( $V_{0}=8.89 \mathrm{~m}^{3} / \mathrm{kg}$ ) and the combustion products of $\mathrm{N}_{2}$, CO , and $\mathrm{CO}_{2}$, and denoting by m the fraction of carbon that has been converted to CO , it is found that

$$
\begin{equation*}
\alpha(z)=\frac{1}{1+\frac{79}{21} \frac{0.5 \mathrm{CO}}{N_{2}}}=\frac{1}{1+\frac{79}{21} \frac{1.866 \cdot 0.5 m}{0.79 \alpha(z) V_{0}}}=\frac{1}{1+\frac{0.5 m}{\alpha(z)}} . \tag{13}
\end{equation*}
$$

Hence $m=2(1-\alpha(z))$.
If the heat of burnup of carbon to $\mathrm{CO}_{2}$ is denoted by $\mathrm{Q}_{1}=393.77 \mathrm{~kJ} / \mathrm{mole}$, and the corresponding quantity for burnup to $C O$ by $Q_{2}=110.61 \mathrm{~kJ} / \mathrm{mole}$, the following expression is obtained for the chemically incomplete combustion as a function of $\alpha(z)$

$$
\begin{equation*}
q_{3}=m\left(1-\frac{Q_{1}}{Q_{2}}\right)=1.44(1-\alpha(z)) \tag{14}
\end{equation*}
$$

This formula, of course, is meaningful for $\alpha \leqslant 1$. Taking into account that $\alpha(z)=1 /(2 \psi(z))$, the extent of chemically incomplete combustion may be expressed in terms of $\psi(z)$

$$
\begin{equation*}
q_{3}=1.44\left(1-\frac{0.5}{\psi(z)}\right) \tag{15}
\end{equation*}
$$

The results of calculation from Eq. (14) may differ somewhat from experimental values of qu, not only because they are obtained for pure carbon, but also because of the possible oxygen content in the combustion products containing CO, since Eq. (14) is actually based on the concentration of excess carbon monoxide, for the burnup of which there is insufficient oxygen in the gas.

The total amount of fuel carried away in the form of incomplete-combustion products is equal to the product of the chemically incomplete burnup and the amount of gasified fuel

$$
\begin{equation*}
Q_{3}=\int_{0}^{l} q_{3} f(z(x)) d x . \tag{16}
\end{equation*}
$$

The average over all the gasified fuel of the chemically incomplete combustion is $\bar{q}_{3}=Q_{3} / j$. Using Eqs. (4), (12), (14), and (16), it is found that

$$
\begin{equation*}
\bar{q}_{3}=1.44 \bar{\alpha} \int_{0}^{\tilde{x}_{p}} \psi(z(x)) d x-0.72 \bar{\alpha} \tilde{x}_{p} \tag{17}
\end{equation*}
$$

Integration here is taken only up to the coordinat $\tilde{x}_{p}$ at which $q_{9}$ becomes equal to zero - or $\psi(z)=0.5$ in Eq. (15) - because when $\tilde{x}>\tilde{x}_{p}$ Eqs. (14) and (15) are meaningless. This coordinate is found from Eq. (9), in which $z$ is replaced by the value $z_{p}$, corresponding to $\psi(z)=0.5$.

In Fig. 1, the distribution of fuel concentration over the length of the combustion chamber is shown, as calculated on a computer from Eq. (9), where Eq. (5) is used to approximate $\psi(z)$. The quantity $z_{0}$ is determined from Eq. (8), in which, to simplify the calculations, it is assumed that $z_{Z}=0$, i.e., in fact, the combustion chamber of finite length is regarded as the corresponding section of an infinite chamber. As shown by computer calculations using Eq. (9), the assumption made has no serious influence on the form of the given dependences. Thus, for $\tilde{D}=1.07$ and $\alpha=1.2$, the fuel concentration $z$ is reduced from 0.38 at $\tilde{x}=0$ to 0.00038 at $\tilde{x}=1$. A somewhat smaller reduction in concentration is noted at low initial fuel concentrations. For example, for $\tilde{D}=8.93$ and $\alpha=1.2$, the fuel concentration at $\tilde{x}=0$ is 0.07 and that at $\tilde{x}=1$ is 0.0022 . Nevertheless, in this case also, $z_{\eta} \ll z_{0}$, i.e., assuming that $z_{\ell}=0$ does not introduce significant distortion into the calculation of the concentration distribution over the length of the chamber.

It is evident from Fig. 1 that when $\tilde{D}<7$ the fuel concentration over the length of the chamber changes in an extremely nonuniform manner. Here, as is evident from Fig. 2, which is plotted from Eq. (17), the magnitude of the chemically incomplete combustion also increases significantly. It is interesting to note that, in accordance with Eq. (5), the function $\psi(z)$


Fig. 1. Distribution of fuel concentration $z$ over the length of the combustion chamber; $z, \tilde{D}, \tilde{x}$ are dimensionless quantities.
Fig. 2. Dependence of the chemically incomplete combustion $q_{3}$ (\%) on the reduced diffusion coefficient $\tilde{D}: 1$ ) $\alpha=1.0 ; 2$ ) 1.18; 3) 1.34 .
has a maximum equal to 0.65 , and the maximum value of $q_{3}$ does not exceed $1.44(1-0.5 / \psi(z))=$ 0.33. For other fuels and combustion conditions, these quantities are difficult, of course.

Calculation from Eq. (5) shows that the value of $z_{p}$ corresponding to $\psi\left(z_{p}\right)=0.5$ is 0.0424 . This means that, in the specific conditions given, complete burnup without excess oxygen in the products released from the bed (i.e., $\alpha=1$ ) is achieved at a fuel concentration in the bed of $4.24 \%$ (by mass). At high concentrations, chemically incomplete combustion is unavoidable; at lower concentrations, the combustion products will contain excess air. The cross section $\tilde{x}_{p}$ in which this fuel concentration is reached depends on the parameters $\tilde{D}$ and $\alpha$ of the process.

In the above calculations, no account is taken of so-called mechanically incomplete combustion, associated with the fuel content in the dust released from the bed, which increases with increase in carbon concentration in the bed and decrease in $\alpha$. In these conditions, the most acceptable value of $q_{3}$ is no more than $1 \%$. It is evident from Fig. 2 that the corresponding value of D is $\tilde{\mathrm{D}}>10$.

For rough calculations, the Todes formula is used [3]

$$
\begin{equation*}
D=\frac{1}{60} \sqrt{L^{3} g} . \tag{18}
\end{equation*}
$$

Here $L$ is the minimum dimension determining the scale of the circulational vortex (in the present case, the bed height $h$ ). Substituting $D$ from Eq. (18) into the expression for $\tilde{D}$ yields

$$
\tilde{D}=\frac{V_{0} \rho \sqrt{g h}}{60 w}\left(\frac{h}{l}\right)^{2} .
$$

Assuming $\mathrm{L}=\mathrm{h}=0.5 \mathrm{~m}, \rho=10^{3} \mathrm{~kg} / \mathrm{m}^{3}, \mathrm{~V}_{0}=8.89 \mathrm{~m}^{3} / \mathrm{kg}$, as for pure carbon, and setting $\mathrm{w}=1$ $\mathrm{m} / \mathrm{sec}$ (remember that the velocity reduced to normal conditions appears in the formula for $\tilde{D}$ ), it is found that $Z=2 \mathrm{~m}$ when $\tilde{\mathrm{D}}=20$. Since in boilers with a fluidized bed the tube bundles are arranged sufficiently close together, the diffusion coefficient must be taken for a free, but for an organized bed, i.e., an order of magnitude lower than that used for the Todes estimates. This reduces $l$ by a factor of $\approx 3$. Thus, even in the one-dimensional case, i.e., in loading fuel over the whele front, normal operation of the boiler may be ensured at a length of no more than 1 m . There is no possibility here of detailed analysis of the problem with point delivery of fuel. It is clear that nonuniformity of the fuel distribution over the bed cross section will be even larger in this case, since the experimental recommendation obtained, that one input point be established per $\mathrm{m}^{2}$ of bed area, does not seem excessive. Loading fuel by this means is inappropriate in a large industrial furnace. To find the optimal construction of a boiler device, the following avenues seem promising (Fig. 3):

1) uniform loading of fuel downward through the gas-distributor grid (Fig. 3a) using a stoker or by any other means;


Fig. 3. Organization of furnace process.
2) loading the fuel from the front with organization of the directed motion of inert material and fuel along the furnace and continuous return of inert material to the front of the furnace (Fig. 3b);
3) two-stage combustion, in which a high concentration of fuel is maintained in the furnace, ensuring that the excess-air coefficient is less than unity (Fig. 3c).

Detailed discussion of these methods falls outside the scope of the present work.

## NOTATION

C, fuel concentration in bed, $\mathrm{kg} / \mathrm{m}^{3} ; \mathrm{CO}, \mathrm{CO}_{2}, \mathrm{O}_{2}, \mathrm{~N}_{2}$, concentration of carbon monoxide, carbon dioxide, oxygen, and nitrogen in the combustion products, vol. \%; D, effective diffusion coefficient, $\mathrm{m}^{2} / \mathrm{sec} ; \tilde{\mathrm{D}}$, reduced diffusion coefficient; j , flux of fuel supplied to the bed, $\mathrm{kg} / \mathrm{m} \cdot \mathrm{sec} ; \mathrm{h}, \mathrm{fluidized-bed}$ height, $\mathrm{m} ; \mathrm{m}$, fraction of carbon reacting to form CO ; 2 , com-bustion-chamber length, $m ; L$, dimension of circulational vortex, $m ; V_{0}$, theoretically necessary volume of air for complete burnup of 1 kg of fuel, $\mathrm{m}^{3} / \mathrm{kg} ; \mathrm{Q}_{1}, Q_{2}$, heat of combustion of carbon to form $\mathrm{CO}_{2}$ and CO , respectively, $\mathrm{kJ} / \mathrm{mole} ; \mathrm{Q}_{3}$, extent of chemically incomplete combustion, $\mathrm{kg} / \mathrm{sec} ; \mathrm{q}_{3}$, relative magnitude of chemically incomplete combustion; $w$, rate of fluidization; $x$, coordinate, $m ; \tilde{x}$, dimensionless coordinate; $z$, relative fuel concentration, $\mathrm{kg} / \mathrm{kg}$; $\alpha(z)$, excess-air coefficient; $\bar{\alpha}$, mean excess-air coefficient; $\rho$, fluidized-bed density, $\mathrm{kg} / \mathrm{m}^{3}$; $\psi(z)$, a dimensionless complex.

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